

# Gravity duals for non-relativistic CFTs

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We attempt to generalize the AdS/CFT correspondence to non-relativistic conformal field theories which are invariant under Galilean transformations. Such systems govern ultracold atoms at unitarity, nucleon scattering in some channels, and more generally, a family of universality classes of quantum critical behavior. We construct a family of metrics which realize these symmetries as isometries. They are solutions of gravity with negative cosmological constant coupled to pressureless dust. We discuss realizations of the dust, which include a bulk superconductor. We develop the holographic dictionary and find two-point correlators of the correct form. A strange aspect of the correspondence is that the bulk geometry has two extra noncompact dimensions.

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## 1. Introduction

Many attempts have been made to use the AdS/CFT correspondence [1,2,3] to study systems realizable in a laboratory. One does not yet have a holographic dual matching the precise microscopic details of any such system and is therefore led to try to match the universality class of the system. In general, physics in the far infrared is described by a (sometimes trivial) fixed point of the renormalization group. It has been argued that the associated zero-temperature conformal field theory (CFT) controls a swath of the finite-temperature phase diagram, namely the region in which the temperature is the only important scale, and no dimensionful couplings have turned on (see *e.g.* [4]). In trying to use gauge/string duality to study such questions, it seems urgent, then, to match the symmetries to those of the system of interest.

The AdS/CFT correspondence so far gives an effective description of relativistic conformal field theories at strong coupling. Not many of these are accessible experimentally.

However, there are many *non-relativistic* conformal field theories which govern physical systems. Such examples arise in condensed matter physics [4], atomic physics [5], and nuclear physics [6]. In the first situation, these are called ‘quantum critical points’; this term refers to a second-order phase transition that is reached not by varying the temperature, but by varying some coupling constants at zero temperature.

A particularly interesting example where exquisite experimental control is possible is the case of cold fermionic atoms at unitarity (*e.g.* [5] and references therein and thereto). These are scale invariant in the following sense. The interactions are tuned (by manipulating the energy of a two-body boundstate to threshold) to make the scattering length infinite (hence the system is strongly coupled). The material is dilute enough that the effective range of the potential can be treated as zero. While the mass of the atoms is a dimensionful quantity, the dependence of the energy and other physical quantities on that mass is determined by the symmetry algebra. A recent paper studying the constraints from Galilean conformal symmetry on such systems is [7] (other recent theory references include [8]).

In this brief note, we set out to find a bulk dual of non-relativistic CFTs, analogous to the AdS gravity description of relativistic CFTs, at strong coupling. We approach this question by considering the algebra of generators of the non-relativistic conformal group, which appears in [7] (related work includes [9]). We seek a solution of string theory (in its low-energy gravity approximation) whose asymptotic boundary symmetry group is not the

Poincaré symmetry group (*i.e.* Lorentz and translations), but rather Galilean invariance and translations. We also demand a non-relativistic version of scale invariance and, when possible, special conformal transformations.

Clearly we will have to introduce some background energy density to find such a solution (*i.e.* it will not be just a solution of gravity with a cosmological constant). Holographically, this is because the theories we are trying to describe arise in general (perhaps not always) from relativistic microscopic theories (not necessarily conformal) upon by adding in some background of Stuff (which fixes a preferred reference frame) and then taking some limit focussing on excitations with small velocities in this frame.

An analogy which may be useful is the following. The plane-wave symmetry group is a contraction of the AdS isometry group; the plane wave solution arises from AdS in considering a limit focussing on the worldline of a fast-moving particle. So it might be possible to take some limit of AdS analogous to the plane wave limit to find our solution. The limit involved would be like the Penrose limit, but replacing the single particle worldline with a uniform background number density. We leave such a derivation from a more microscopic system for the future and proceed to guess the end result.

Scale invariance can be realized in non-relativistic theories in a number of ways. One freedom is the relative scale dimension of time and space, called the ‘dynamical exponent’  $z$  (see *e.g.* [10,11]). The familiar Schrödinger case has  $z = 2$ , and the conformal algebra for this case is described in [7]. Fixed points with  $z \neq 2$  can be Galilean invariant, too. We will find a metric with this symmetry group as its isometry group for each value of  $z$ .<sup>1</sup> To be specific, the relevant algebra is:

$$\begin{aligned}
[M_{ij}, N] &= [M_{ij}, D] = 0, & [M_{ij}, P_k] &= i(\delta_{ik}P_j - \delta_{jk}P_i), & [M_{ij}, K_k] &= i(\delta_{ik}K_j - \delta_{jk}K_i) \\
[M_{ij}, M_{kl}] &= i(\delta_{ik}M_{jk} - \delta_{jk}M_{il} + \delta_{il}M_{kj} - \delta_{jl}M_{ki}) \\
[P_i, P_j] &= [K_i, K_j] = 0, & [K_i, P_j] &= i\delta_{ij}N, & [D, P_i] &= iP_i, & [D, K_i] &= (1-z)iK_i \\
[H, N] &= [H, P_i] = [H, M_{ij}] = 0, & [H, K_i] &= -iP_i, & [D, H] &= zH, & [D, N] &= i(2-z)N.
\end{aligned}
\tag{1.1}$$

Here  $i, j = 1..d$  label the spatial dimensions.  $M_{ij}$  generate spatial rotations, and the first two lines just state the properties of the various generators under spatial rotations.  $P_i$  are momenta,  $K_i$  generate Galilean boosts,  $N$  is a conserved rest mass or particle number, and

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<sup>1</sup> Gravity duals of related but distinct Lifshitz points (with time reversal invariance) are being studied by S. Kachru, X. Liu, and M. Mulligan [12]. We thank them for discussions.

$D$  is the dilatation operator. That the particle number can be conserved is a consequence of the possibility of absence of antiparticles in a non-relativistic theory<sup>2</sup>.

For the special case  $z = 2$ , there is an additional special conformal generator  $C$  which enjoys the algebra:

$$[M_{ij}, C] = 0, \quad [K_i, C] = 0, \quad [D, C] = -2iC, \quad [H, C] = -iD.$$

The metrics we find are also invariant under the combined operations of time-reversal and charge conjugation.

In the next section we write down a metric with this isometry group.<sup>3</sup> It is sourced by a stress tensor describing a negative cosmological constant plus dust. After demonstrating the action of the isometry group, we provide answers to the important question: why doesn't the dust collapse due to the gravitational attraction? In section three, we generalize the AdS/CFT dictionary to extract information about the field theory dual, including anomalous dimensions and Green's functions. In section four, we highlight some of the many open questions raised by this construction.

## 2. Geometry

After all this talk, here, finally is the metric:

$$ds^2 = L^2 \left( -\frac{dt^2}{r^{2z}} + \frac{d\vec{x}^2 + 2d\xi dt}{r^2} + \frac{dr^2}{r^2} \right). \quad (2.1)$$

Here  $\vec{x}$  is a  $d$ -vector, and  $z$  is the advertised dynamical exponent. The coordinate  $\xi$  is something new whose interpretation we will develop below.

The metric (2.1) is nonsingular. As is manifest in the chosen coordinates, it is conformal to a pp-wave spacetime<sup>4</sup>.

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<sup>2</sup> That is to say, the dual field theories that we are describing must not have particle production. It is of course possible for non-relativistic theories to nevertheless contain both particles and antiparticles, as indeed do the systems under study in [12], which, unlike ours, are invariant under  $t \rightarrow -t$ .

<sup>3</sup> Earlier work on geometric realizations of the Schrödinger group include [13] and references therein, wherein related metrics are called Bargmann-Einstein structures.

<sup>4</sup> Note added: In the case  $d = 2, z = 3$ , this metric has appeared previously as the near-horizon limit of D3-branes in the presence of lightlike tensor fields [14]; its causal structure was studied in [15]. The sources present in those solutions do not preserve the rotations  $M_{ij}$ .

## 2.1. Symmetries

We shall show that the isometries of the above metric comprise the  $d + 1$ -dimensional non-relativistic dilatation group with algebra (1.1). It is clear that the above metric is invariant under translations in  $\vec{x}, t$  and under rotations of  $\vec{x}$ . It is also invariant under the scale transformation

$$x' = \lambda x, \quad t' = \lambda^z t, \quad r' = \lambda r, \quad \xi' = \lambda^{2-z} \xi \quad .$$

It is invariant under the Galilean boost

$$\vec{x}' = \vec{x} - \vec{v}t \tag{2.2}$$

if we assign the following transformation to the coordinate  $\xi$ :

$$\xi' = \xi + \frac{1}{2}(2\vec{v} \cdot \vec{x} - v^2 t) \quad . \tag{2.3}$$

In the special case  $z = 2$ , special conformal transformations act as

$$\vec{x}' = \frac{\vec{x}}{1 + ct}, \quad t' = \frac{t}{1 + ct}, \quad r' = \frac{r}{1 + ct}, \quad \xi' = \xi + \frac{c}{2} \frac{\vec{x} \cdot \vec{x} + r^2}{1 + ct} \quad .$$

To understand the transformation rule for  $\xi$ , let us define the transformations  $T_1$  and  $T_2$ , such that

$$T_1 : (x, t) \rightarrow \left( \frac{x}{t}, \frac{1}{t} \right), \quad T_2 : (x, t) \rightarrow (x, t + c) \quad ; \tag{2.4}$$

$T_1$  is an inversion and  $T_2$  is a time-translation. The special conformal transformation  $S$  can be written as  $S = T_1 T_2 T_1$ .

One can check that the vector fields generating these isometries have Lie brackets satisfying the algebra (1.1).<sup>5</sup> The only non-obvious identification is that the rest mass is generated by  $N = i\partial_\xi$ . Note that although the metric is not invariant under simple time reversal,  $t \rightarrow -t$ , it is invariant under the combined operation

$$t \rightarrow -t, \quad \xi \rightarrow -\xi, \tag{2.5}$$

which, given the interpretation of the  $\xi$ -momentum as rest mass, can be interpreted as the composition,  $\mathcal{CT}$ , of charge conjugation and time-reversal.

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<sup>5</sup> We believe that no  $d + 1$ -dimensional metric can realize this isometry group.

## 2.2. Einstein's equations

The stress tensor sourcing the metric in equation (2.1) consists of a negative cosmological constant<sup>6</sup>  $\Lambda = -10$  plus a pressureless ‘dust’, with constant density  $\mathcal{E}$ :

$$T_{ab} = -\Lambda g_{ab} - \mathcal{E} \delta_a^0 \delta_b^0 g_{00}.$$

For  $d = 3$ , we find that the density is  $\mathcal{E} = (2z^2 + z - 3) L^{-2}$ .

In the following subsection, we consider what matter can produce this stress tensor. Our answers to this question do not affect our later calculations, except to suggest what fields should be present in the bulk, and to demonstrate that the metric (2.1) is physically sensible.

## 2.3. Dust

We observe that because of the non-stationary form of the metric, the stress tensor for an electric field in the radial direction  $F_{rt}$  has only a 00 component. Maxwell’s equation in the bulk, coupled to a background current  $j^a$  is

$$\frac{1}{\sqrt{g}} \partial_a (\sqrt{g} F^{ab}) = j^b.$$

To produce an electric field in the  $r$  direction, we consider a nonzero  $j^\xi$  of the form

$$j^\xi = \rho_0 r^\alpha. \tag{2.6}$$

Gauss’ law implies that the electrostatic potential is

$$A_0(r) = \frac{\rho_0}{(\alpha - 2)(\alpha - d - 2)} r^{\alpha - 2}. \tag{2.7}$$

The current and gauge field are therefore related by the London equation  $j_a = m_A^2 A_a$ , with  $m_A^2 = \frac{z(z+d)}{L^2}$ .<sup>7</sup>

Having gained this insight, we observe that our metric is sourced by the ground state of an Abelian Higgs model in its broken phase. The model

$$S = \int d^{d+3} x \sqrt{g} \left( -\frac{1}{4} F^2 + \frac{1}{2} |D\Phi|^2 - V(|\Phi|^2) \right)$$

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<sup>6</sup> We have set  $8\pi G_N = 1$ . The indices  $a, b$  run over all  $d + 3$  dimensions of the bulk.

<sup>7</sup> Recent holographic studies of spontaneously broken global symmetries of the boundary theory include [16] and (at finite temperature) [17].

with  $D_a \Phi \equiv (\partial_a + ieA_a)\Phi$ , with a Mexican-hat potential

$$V(|\Phi|^2) = g(|\Phi|^2 - v^2)^2 + \Lambda$$

produces the correct dust stress tensor for arbitrary  $g$ , as long as  $e^2 v^2 = m_A^2 = \frac{z(z+d)}{L^2}$  as above<sup>8</sup>. The parameter  $\rho_0$  in the solution for the gauge field is determined by the Einstein equation to be  $\rho_0^2 = \mathcal{E} \frac{z(z+d)^2}{2z+d} L^4$ . It is tempting to relate the phase of  $\Phi$  to that of the boundary wavefunction. We hope in the future to use this theory with finite  $g$  to describe the vortex lattice formed by rotating the cold-atom superfluids.

The fact that the metric (2.1) is sourced by a reasonable class of physical systems with stable ground states suggests that it need have no intrinsic instabilities.

### 3. Correspondence

Consider a scalar field in the background (2.1), with action

$$S = -\frac{1}{2} \int d^{d+3}x \sqrt{g} (\partial_\mu \phi \partial_\nu \phi g^{\mu\nu} + m^2 \phi^2) ,$$

Here  $d^{d+3}x \equiv d^d \vec{x} dt d\xi dr$ . The usual argument [2] that  $h_{xy}(k_z)$  satisfies the same equation of motion as a massless scalar goes through, and so we can take the  $m = 0$  case as a proxy for this component of the metric fluctuations. In the metric (2.1), we have  $\sqrt{g} = r^{-(d+3)}$  for any  $z$ . The wave equation in this background has the form

$$0 = \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \phi) - m^2 \phi = \left[ r^{d+3} \partial_r \left( \frac{1}{r^{d+1}} \partial_r \right) + r^2 \left( 2\partial_\xi \partial_t + r^{2-2z} \partial_\xi^2 + \vec{\nabla}^2 \right) - m^2 \right] \phi.$$

Translation invariance allows us to Fourier decompose in the directions other than  $r$ ,  $\phi(r) = e^{i\omega t + i\vec{k} \cdot x + i l \xi} f_{\omega, \vec{k}, l}(r)$ . Then:

$$\left[ -r^{d+3} \partial_r \left( \frac{1}{r^{d+1}} \partial_r \right) + r^2 (2l\omega + \vec{k}^2) + r^{4-2z} l^2 + m^2 \right] f_{\omega, \vec{k}, l}(r) = 0.$$

To find the generalization of the gauge/gravity dictionary to this case, we first study asymptotic solutions of this equation near the boundary. Writing  $f \propto r^\Delta$ , we find for  $z \leq 2$ :

$$\Delta_\pm = 1 + \frac{d}{2} \pm \sqrt{\left(1 + \frac{d}{2}\right)^2 + m^2 + \delta_{z,2} l^2} . \quad (3.1)$$

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<sup>8</sup> Our metric is therefore a solution of the same system studied in the first and last references in [17], with very different asymptotics.

In the case  $z > 2$ , there is no scaling solution near the boundary; the  $\xi$ -momentum dominates the boundary behavior, and the solutions vary with  $r$  faster than any power of  $r$ . It would be interesting to understand a possible meaning of the asymptotic solutions which do arise, and to interpret the critical nature of  $z = 2$  in terms of the boundary theory. As usual, these exponents  $\Delta_{\pm}$  can be interpreted as scaling dimensions of spin-zero boundary operators in the boundary theory. We look forward to extending this analysis to the fluctuations of fields of other spin. Instead we will proceed to compute correlation functions of these scalar operators.

### 3.1. Correlators

For definiteness, we focus on the familiar, critical  $z = 2$  case with  $d = 3$  spatial dimensions. In that case, the momentum in the  $\xi$ -direction simply adds to the mass of the bulk scalar, *i.e.* they appear in the combination  $l^2 + m^2$ . In this case, the wave equation becomes

$$\left[ -r^6 \partial_r \left( \frac{1}{r^4} \partial_r \right) + r^2 (2l\omega + \vec{k}^2) + l^2 + m^2 \right] f_{\omega, \vec{k}, l}(r) = 0$$

and is solved by

$$f_{\omega, \vec{k}, l}(r) = A r^{5/2} K_{\nu}(\kappa r)$$

where  $K$  is a modified Bessel function,  $A$  is a normalization constant, and<sup>9</sup>

$$\nu = \sqrt{\left(\frac{5}{2}\right)^2 + l^2 + m^2}, \quad \kappa^2 = 2l\omega + \vec{k}^2.$$

As usual in AdS/CFT, we choose  $K$  over  $I$  because it is well-behaved near the horizon,  $K(x) \sim e^{-x}$  at large  $x$ .

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<sup>9</sup> For general  $d$ , with  $z = 2$ , the solution is

$$f_{\omega, \vec{k}, l}(r) = A r^{1+\frac{d}{2}} K_{\nu}(\kappa r)$$

with

$$\nu = \sqrt{\left(1 + \frac{d}{2}\right)^2 + M^2}, \quad \kappa^2 = 2l\omega + \vec{k}^2, \quad M^2 \equiv m^2 + \delta_{z,2} l^2.$$



To compute correlators, we introduce a cutoff near the boundary at  $r = \epsilon$ , and normalize  $f_\kappa(\epsilon) = 1$  (so  $A = \epsilon^{-5/2} K_\nu(\kappa\epsilon)^{-1}$ ). The usual prescription goes through, leaving an on-shell bulk action of the form

$$S[\phi_0] = \frac{1}{2} \left[ \int d^{d+2} X \sqrt{g} g^{rr} \phi(X) \partial_r \phi(X) \right]_{r=\epsilon}$$

where  $X \in \{\vec{x}, t, \xi\}$  are the coordinates on the boundary. This evaluates to

$$S[\phi_0] = \frac{1}{2} \int dp \phi_0(-p) \mathcal{F}(\kappa, \epsilon) \phi_0(p)$$

with the ‘flux factor’

$$\mathcal{F}(\kappa, \epsilon) = \lim_{r \rightarrow \epsilon} \sqrt{g} g^{rr} f_\kappa(r) \partial_r f_\kappa(r) = \sqrt{g} g^{rr} \partial_r \ln \left( r^{\frac{2+d}{2}} K_\nu(\kappa r) \right) \Big|_{r=\epsilon} .$$

Using the expansion for small  $x$

$$K_\nu(x) \simeq 2^{\nu-1} \Gamma(\nu) x^{-\nu} \left( 1 + \dots - \left( \frac{x}{2} \right)^{2\nu} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)} \right)$$

and ignoring contact terms, this is

$$\mathcal{F}(\kappa, \epsilon) = -\frac{\Gamma(1-\nu)}{\Gamma(\nu)} \frac{1}{2\epsilon^5} \left( \frac{\kappa\epsilon}{2} \right)^{2\nu} .$$

This gives

$$\langle \mathcal{O}_1(\omega, \vec{k}) \mathcal{O}_2(\omega', \vec{k}') \rangle \propto \delta(k + k') \frac{\Gamma(1-\nu)}{\Gamma(\nu)} \frac{1}{2\epsilon^5} \left( \frac{(2l\omega + \vec{k}^2)\epsilon^2}{4} \right)^\nu$$

(in the case  $\phi = h_{xy}$ ,  $\mathcal{O} = T_{xy}$ ). We defer a more careful treatment of holographic renormalization to future work. Fourier transforming to position space, this is

$$\langle \mathcal{O}_1(x, t) \mathcal{O}_2(0, 0) \rangle \propto \frac{\Gamma(1-\nu)}{\Gamma(\nu)} \delta_{\Delta_1, \Delta_2} \theta(t) \frac{1}{|\epsilon^2 t|^\Delta} e^{-ilx^2/2|t|}$$

where we used translation invariance to put the second operator at  $(\vec{x}, t) = (\vec{0}, 0)$ ,  $\theta$  is a step function, and  $\Delta_{i,j}$  are the scaling dimensions of the operators that we found in (3.1). This is the form one expects for the correlators in a Galilean-invariant conformal field theory in  $d$  spatial dimensions with dynamical exponent 2. It will be interesting to extend this calculation to other values of  $z$ .

## 4. Discussion

In discussions of AdS/CFT, one often hears that the CFT ‘lives at the boundary’ of the bulk spacetime. The spacetime (2.1) is conformal to a pp-wave spacetime, and hence has a boundary which is one dimensional – for  $z > 1$ ,  $g_{tt}$  grows faster than the other components at small  $r$ . While one might be tempted to speculate that this means that the dual field theory is one dimensional, there is no clear evidence for this statement (in the usual case, only the UV of the field theory can really be said to live at the boundary of the AdS space), and indeed we have nevertheless used it to compute correlators which look like those of a  $d$ -dimensional nonrelativistic field theory.

Perhaps the most mysterious aspect of this new correspondence is not of too few dimensions, but of too many<sup>10</sup>. The  $\xi$  direction is an additional noncompact dimension of the bulk geometry besides the usual radial direction, which seems to be associated to the conserved rest mass<sup>11</sup>. We have computed correlators at fixed  $l$ ; this is reasonable since the theory is non-relativistic and so a sector with definite  $l$  can be closed (unlike in a relativistic theory where these sectors unavoidably mix by production of back-to-back particles and antiparticles). At the special value of the dynamical exponent  $z = 2$ , varying  $l$  affects not just the units in which energy is measured, but the values of the critical exponents (see equation (3.1)).

We would like to think of these solutions as scaling limits which zoom in on small fluctuations about particular states of relativistic theories (possibly CFTs), where a preferred rest frame is fixed by some density of Stuff. It would be nice to see our metric arise as such a contraction of an asymptotically AdS solution. Such an identification may help find the finite-temperature geometry.

A word of caution in this direction is in order: at large R-charge but low temperature, there is an instability in supersymmetric theories to Bose condensation, because there are inevitably scalar fields that carry R-charge. A system with symmetries under which only fermions are charged (such as [21]) could be useful, and would seem more likely to be related to cold fermions away from the fixed point.

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<sup>10</sup> Other codimension-two holographic relations include [18], where one of the missing dimensions is time, and [19], which can be understood by a Kaluza-Klein decomposition with discrete momenta.

<sup>11</sup> The similarity with the plane wave appears here, too: the plane wave describes a relativistic system in the infinite momentum frame, where there is also no particle production [20]. We thank Allan Adams for this observation.

Many questions remain. We have computed the correlator  $\langle T_{xy}T_{xy} \rangle$ . Does this plus the dynamical exponent completely determine the  $TT$  correlator, or are there independent central charges in the non-relativistic theory?

One interesting observable, which has been *measured* as well as computed numerically for cold-fermion systems is the analog of the famous  $3/4$  in the  $\mathcal{N} = 4$  SYM theory, namely the ratio of the energy density at unitarity to the energy density of the corresponding free theory ( $\xi$  in [22] and [8], proportional to  $\beta$  in [23]).<sup>12</sup> We will need to learn to compute the analog of the ADM mass for spacetimes with the asymptotium of (2.1).

Once the finite-temperature solution with these asymptotics is found, there will be many more interesting things to compute. Of course, it will be nice to check the universality of  $\frac{\eta}{s}$  [24], obtainable from the correlator we have studied, at nonzero temperature. As a possible but quite ambitious example of a qualitative consequence of the non-relativistic microscopic constituents in these systems we offer the following. It has been noted [*e.g.* 25] that the plasma made by putting any relativistic CFT at finite temperature is quite compressible. The compressibility is known to have an important effect on the turbulence cascade [26]; it can even change its direction. It is therefore of interest to have gravity descriptions of less compressible fluids, which these would presumably be.

While this work was being completed, we learned that Dam Son has independently found very closely related results [27].

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